

CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
EET401	ADVANCED CONTROL SYSTEMS	PCC	2	1	0	3

**Preamble:** This course aims to provide a strong foundation on advanced control methods for modelling, time domain analysis, and stability analysis of linear and nonlinear systems. The course also includes the design of feedback controllers and observers.

**Prerequisite:** EET 305 Signals and Systems, EET 302 Linear Control Systems

**Course Outcomes:** After the completion of the course the student will be able to:

CO 1	Develop the state variable representation of physical systems
CO 2	Analyse the performance of linear and nonlinear systems using state variable approach
CO 3	Design state feedback controller for a given system
CO 4	Explain the characteristics of nonlinear systems
CO 5	Apply the tools like describing function approach or phase plane approach for assessing the performance of nonlinear systems
CO 6	Apply Lyapunov method for the stability analysis of physical systems.

**Mapping of course outcomes with program outcomes**

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	-	-	-	-	-	-	-	-	-	2
CO 2	3	3	2	-	-	-	-	-	-	-	-	2
CO 3	3	3	3	-	-	-	-	-	-	-	-	2
CO 4	3	2	-	-	-	-	-	-	-	-	-	2
CO 5	3	3	2	-	-	-	-	-	-	-	-	2
CO 6	3	3	2	-	-	-	-	-	-	-	-	2

**Assessment Pattern:**

Total Marks	CIE marks	ESE marks	ESE Duration
150	50	100	03 Hrs

Bloom's Category	Continuous Assessment Tests		End Semester Examination
	1	2	
Remember (K1)	10	10	20
Understand (K2)	15	15	30
Apply (K3)	25	25	50
Analyse (K4)			
Evaluate (K5)			
Create (K6)			

**End Semester Examination Pattern:** There will be two parts; Part A and Part B.

Part A contains 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions.

Part B contains 2 questions from each module of which student should answer any one. Each question carries 14 marks and can have maximum 2 sub-divisions.

**Course Level Assessment Questions:**

**Course Outcome 1 (CO1)**

1. Derive the state model of an armature controlled DC servo motor. (K2, PO1)
2. Obtain the phase variable representation for the system with  $G(s)=$

$$T(s) = \frac{2s^2 + s + 3}{s^3 + 6s^2 + 11s + 6} \quad (\text{K3, PO1, PO2})$$

3. Problems on deriving the state model of a given electrical circuit. (K2, PO1)
4. Problems on the conversion of Phase variable form to Canonical form. (K3, PO1, PO2)

**Course Outcome 2 (CO2):**

1. Obtain the time response  $y(t)$  of the homogeneous system:

$$\dot{X} = \begin{bmatrix} -1 & 1 \\ -2 & -3 \end{bmatrix} x, \quad y = [1 \quad 1]x \quad \text{and} \quad x(0)^T = [1 \quad 0] \quad (\text{K3, PO1, PO2})$$

2. Determine the transfer function for the system with the state model:

$$\dot{X} = \begin{bmatrix} -2 & 1 \\ -3 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u; \quad y = [1 \quad 1]x \quad (\text{K3, PO1, PO2})$$

3. Determine the controllability of the following state model:

$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad (\text{K3, PO1, PO2, PO3})$$

**Course Outcome 3(CO3):**

1. Design a state feedback controller for the following system such that the closed loop poles are placed at:  $-1 \pm j2$  and  $-12$ .

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & -1 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad (\text{K3, PO1, PO2, PO3})$$

2. Design problems on State observer. (K3, PO1, PO2, PO3)

**Course Outcome 4 (CO4):**

1. Explain the linearization concept and assumptions made referred to Describing Function analysis. (K1, PO1)
2. With suitable characteristics explain the jump resonance phenomena. (K2, PO1, PO2)
3. Differentiate between linear and nonlinear systems referred to:
  - i) frequency response, ii) sustained oscillations. (K2, PO1, PO2)
4. Identify and explain the type of singular points for the following two systems:

$$\text{i) } \dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} X \quad \text{and} \quad \text{ii) } \dot{X} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} X \quad (\text{K3, PO1, PO2})$$

**Course Outcome 5 (CO5):**

1. Problems related to the derivation of describing function of a common nonlinearity. (K2, PO1, PO2)
2. Problems related to application of describing function for analysing the stability of given closed loop system. (K3, PO1, PO2, PO3)
3. Obtain the phase trajectory of the system with  $\ddot{y} + 6\dot{y} + 5y = 0$ , for initial point  $x(0)^T = [1 \ 0.6]$ . Use Isocline method. Also, identify the type of singular point. (K3, PO1, PO2, PO3)

**Course Outcome 6 (CO6):**

1. Use Lyapunov Direct method to determine the value of K such that the given LTI system is stable.

$$\dot{X} = \begin{bmatrix} 0 & K \\ -2 & -1 \end{bmatrix} X \quad (\text{K3, PO1, PO2, PO3})$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -5 \end{bmatrix} X$$

2. Determine the stability of the LTI system with state model:  
(K3, PO1, PO2, PO3)
3. Test stability of the nonlinear system given below, using Lyapunov method.

$$\dot{X} = \begin{bmatrix} -4 & 0 \\ 3x_2^2 & -2 \end{bmatrix} X \quad (\text{K3, PO1, PO2, PO3})$$



**Model Question Paper**

PAGES: 3

QP CODE:

Reg.No: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDULKALAM TECHNOLOGICAL UNIVERSITY**  
**SEVENTH SEMESTER B.TECH DEGREE EXAMINATION**  
**MONTH & YEAR**

Course Code: EET401

Course Name: **ADVANCED CONTROL SYSTEMS****Max. Marks: 100****Duration: 3 Hours****PART A****Answer all Questions. Each question carries 3 Marks**

1 Selecting  $i_1(t) = x_1(t)$  and  $i_2(t) = x_2(t)$  as state variables obtain state equation and output equation of the network shown.

2 Obtain the diagonal canonical representation for the system with the transfer function:

$$T(s) = \frac{s + 2}{s^2 + 0.7s + 0.1}$$

3 Determine the transfer function for the system with state model:

$$\dot{X} = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u; \quad y = [1 \quad 0]x$$

4 Explain any four properties of state transition matrix.

5 Determine the controllability of the following state model:

$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

6 Explain the significance of PBH test for observability.

7 With suitable characteristics explain the jump resonance phenomena in nonlinear systems.

8 Obtain the describing function of deadzone non-linearity.

9 Determine given quadratic form is positive definite or not:

$$V(x) = 10x_1^2 + 4x_2^2 + x_3^2 + 2x_1x_2 - 2x_2x_3 - 4x_1x_3$$

10 Use Lyapunov theorem to determine test stability of the nonlinear system given below.

$$\dot{X} = \begin{bmatrix} -4 & 0 \\ 3x_2^2 & -2 \end{bmatrix} X$$

**PART B****Answer any one full question from each module. Each question carries 14 Marks****Module 1**

11 a) Obtain the phase variable representation for the system with transfer function:

$$T(s) = \frac{2s^2 - 3}{s^3 + 6s^2 + 11s + 6}$$

(7 Marks)

b) Derive the state model of an armature controlled DC servo motor.

(7 Marks)

12 a) Determine the diagonal canonical representation for the system:

$$\dot{X} = \begin{bmatrix} -2 & 1 \\ -3 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u; \quad y = [1 \quad -1]x. \quad (9 \text{ Marks})$$

- b) Explain any four advantages of state model as compared to transfer function model. (5 Marks)

### Module 2

- 13 a) Obtain the unit step response  $y(t)$  of the system

$$\dot{X} = \begin{bmatrix} -1 & 0 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u, \quad y = [1 \quad 1]x. \quad (10 \text{ Marks})$$

- b) Show that eigen values of state models are unique. (4 Marks)

- 14 a) Determine the state transition matrix for the system with state model:

$$\dot{X} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} x \quad (7 \text{ Marks})$$

- b) How do you derive the z transfer function from the state model of a sampled data system? (7 Marks)

### Module 3

- 15 a) Consider a linear system described by the transfer function  $\frac{Y(s)}{U(s)} = \frac{10}{s(s+1)(s+2)}$ . Design a feedback controller with a state feedback so that the closed loop poles are placed at  $-2, -1 \pm j1$ . (10 Marks)

- b) Write short note on reduced order observer. (4 Marks)

- 16 a)  $\dot{x} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & -5 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} u$

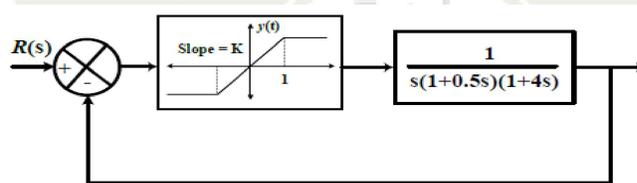
Consider a linear system described by

Design a state observer so that the closed loop poles are placed at  $-4, -3 \pm j1$ . (9 Marks)

- b) With suitable example explain the concept of duality referred to controllability. (5 Marks)

### Module 4

- 17 a) Determine the value of  $K$  for an occurrence of limit cycle. Also determine the amplitude, frequency and stability of limit cycle. (10 Marks)



- b) With relevant characteristics explain any three nonlinearities in electrical systems. (4 Marks)

- 18 a) Obtain the describing function of relay with dead zone nonlinearity. (8 Marks)  
 b) Explain the linearization concept and assumptions made referred to Describing Function analysis. (6 Marks)

### Module 5

- 19 a) A linear second order system is described by the equation:  $\ddot{e} + 2\delta\omega ne' + \omega n^2 e = 0$ , with  $\delta = 0.25$ ,  $\omega n = 1 \text{ rad/sec}$ ,  $e(0) = 1.0$ , and  $\dot{e}(0) = 0$ . Determine the singular point and state the stability by constructing the phase trajectory using the method of isoclines. (11 Marks)  
 b) Identify and explain the type of singular point for the following system:

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} X$$

(3 Marks)

- 20 a) Differentiate between stable and unstable limit cycles. (5 Marks)
- b) Use Lyapunov Direct method to determine the value of K such that the given LTI system is stable.

$$\dot{X} = \begin{bmatrix} 0 & K \\ -2 & -1 \end{bmatrix} X$$

(9 Marks)

## Syllabus

### Module 1

#### State Space Representation of Systems (7 hours)

Introduction to state space and state model concepts- State equation of linear continuous time systems, matrix representation- features- Examples of electrical circuits and dc servomotors. Phase variable forms of state representation- Diagonal Canonical forms- Similarity transformations to diagonal canonical form.

### Module 2

#### State Space Analysis (9 hours)

State transition matrix- Properties of state transition matrix- Computation of state transition matrix using Laplace transform and Cayley Hamilton method.

Derivation of transfer functions from state equations.

Solution of time invariant systems: Solution of time response of autonomous systems and forced systems.

State space analysis of Discrete Time control systems: Phase variable form and Diagonal canonical form representations- Pulse transfer function from state matrix- Computation of State Transition Matrix (problems from 2<sup>nd</sup> order systems only).

### Module 3

#### State Feedback Controller Design (6 hours)

Controllability & observability: Kalman's, Gilbert's and PBH tests.- Duality principle

State feedback controller design: State feed-back design via pole placement technique

State observers for LTI systems- types- Design of full order observer.

### Module 4

#### Nonlinear Systems (7 hours)

Types and characteristics of nonlinear systems- Jump resonance, Limit cycles and Frequency entrainment

Describing function method: Analysis through harmonic linearization- Determination of describing function of nonlinearities.

Application of describing function for stability analysis of autonomous system with single nonlinearity (relay, dead zone and saturation only).

### Module 5

#### Phase Plane and Lyapunov Stability Analysis (8 hours)

Phase plots: Concepts- Singular points – Classification of singular points.

Definition of stability- asymptotic stability and instability.

Construction of phase trajectories using Isocline method for linear and nonlinear systems.

Lyapunov stability analysis: Lyapunov function- Lyapunov methods to stability of nonlinear systems- Lyapunov methods to LTI continuous time systems.

### Text Books:

1. Nagarath I. J. and Gopal M., Control System Engineering, 5/e, New Age Publishers, 2007
2. Ogata K., Modern Control Engineering, 5/e, Prentice Hall of India, 2010.
3. Gopal M, Modern Control System Theory, 2/e, New Age Publishers, 1984
4. Kuo B.C, Analysis and Synthesis of Sampled Data Systems, Prentice Hall Publications, 2012.

### References:

1. Khalil H. K, Nonlinear Systems, 3/e, Prentice Hall, 2002
2. Gibson J.E. Nonlinear Automatic Control, Mc Graw Hill, 1963.
3. Gopal M., Control Systems Principles and Design, 4/e, Tata McGraw Hill, 2012.
4. Slotine J. E and Weiping Li, Applied Nonlinear Control, Prentice-Hall, 1991,
5. Gopal M, Digital Control and State Variable Methods, 2/e, Tata McGraw Hill, 2003
6. Thomas Kailath, Linear Systems, Prentice-Hall, 1980.
7. Ogata K., Discrete Time Control Systems, 2/e, Pearson Education, Asia, 2015

### Course Contents and Lecture Schedule:

No	Topic	No. of Lectures
<b>1</b>	<b>State Space Representation of Systems</b>	<b>(7 hours)</b>
1.1	Introduction to state space and state model concepts- state equation of linear continuous time systems, matrix representation- features -Examples of electrical circuits and dc servomotors	3
1.2	Phase variable forms of state representation- features- controllable and observable companion forms	2
1.3	Diagonal canonical forms of state representation- Diagonal & Jordan forms- features- Similarity transformations to diagonal canonical form	2
<b>2</b>	<b>State Space Analysis</b>	<b>(9 hours)</b>
2.1	State transition matrix- Properties of state transition matrix- Computation of state transition matrix using Laplace transform- Cayley Hamilton method.	2
2.2	Derivation of transfer functions from state equations.	1
2.3	Solution of time invariant systems: Solution of time response of autonomous systems and forced systems	3
2.4	State space analysis of Discrete Time control systems: Phase variable form and Diagonal canonical form representations	2
2.5	Pulse transfer function from state matrix- Computation of State Transition Matrix- (problems from 2 <sup>nd</sup> order systems only)	1
<b>3</b>	<b>State Feedback Controller Design</b>	<b>(6 hours)</b>
3.1	Controllability & observability: Kalman's, Gilbert's and PBH tests- Duality property	2
3.2	State feedback controller design: State feed-back design via pole placement technique	2

3.3	State observers for LTI systems- Full order and reduced order observers- Design of full order observer design	2
<b>4</b>	<b>Nonlinear Systems</b>	<b>(7 hours)</b>
4.1	Types of nonlinear systems- characteristics of nonlinear systems- peculiar features like Jump resonance, Limit cycles and Frequency entrainment	2
4.2	Describing function Method: Analysis through harmonic linearisation	1
4.3	Determination of describing function of nonlinearities	2
4.4	Application of describing function for stability analysis of autonomous system with single nonlinearity (relay, dead zone and saturation only).	2
<b>5</b>	<b>Phase Plane and Lyapunov Stability Analysis</b>	<b>(8 hours)</b>
5.1	Phase plots: Concepts- Singular points - Classification of singular points.	1
5.2	Construction of phase trajectories using Isocline method for linear and nonlinear systems	2
5.3	Definition of stability- asymptotic stability and instability	1
5.4	Lyapunov stability analysis: Lyapunov function- Lyapunov methods to stability of nonlinear systems	2
5.5	Lyapunov methods to LTI continuous time systems.	2

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